Title of the Research:

A study on inverse ordered semigroups

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SYNOPSIS

Keywords : ordered semigroup, right inverse, inverse, Clifford.

1 Introduction

The notion of inverse semigroups was first introduced by V. V. Wagner in a short note published in 1952 [21], and independently by Gordon Preston in 1954 [14], [15] and [16]. Clifford [3], [4] and Rees [17], [18] have done noticeable contributions to inverse semigroup theory. Inverse semigroups were introduced as part of the legacy of two nineteenth century mathematical enter prises: Klein's Erlanger Programme and Lie's theory of infinite continuous groups.

An initiative was taken by T. Saito to study inverse ordered semigroups and he endowed order with inverse semigroups. He has done an enormous job for our better understanding of such semigroups. But he has studied the characteristic and behavior of order on an inverse semigroup. This does not match with our motivation to study inverse ordered semigroups where the inverse arises out of both the order and the associative binary operation.

2 Brief History of Ordered Semigroup:

An ordered semigroup is a partiality ordered set $(S, \cdot, \leq)$, and at the same time a semigroup $(S, \cdot)$ such that for all $a,\ b,\ x \in S\ a \leq b$ implies $xa \leq xb$ and $ax \leq bx$. It is denoted by $(S, \cdot, \leq)$.

Throughout this article, unless stated otherwise, S stands for an ordered semigroup and We assume that S does not contain the zero element. An equivalence relation ρ is called left (right) congruence if for $a, b, c \in S\ a\ \rho\ b$ implies $ca\ \rho\ cb$ ($ac\ \rho\ bc$). By a congruence we mean both left and right congruence.

A congruence ρ is called semilattice congruence on S if for all $a, b \in S,\ a\ \rho\ a^2$ and $ab\ \rho\ ba$. By a complete semilattice congruence we mean a semilattice congruence σ on S such that for $a, b \in S,\ a \leq b$ implies that $a\sigma ab$. The ordered semigroup S is called complete semilattice of subsemigroups of type τ if there exists a complete semilattice congruence ρ such that $(x)_\rho$ is a type τ subsemigroup of S. Equivalently: There exists a semilattice Y and a family of subsemigroups $\{S\}_{\alpha \in Y}$ of type τ of S such that:

1. $S_\alpha \cap S_\beta = \phi$ for any $\alpha,\ \beta \in Y$ with $\alpha \neq \beta$,
2. $S = \bigcup_{\alpha\ \in\ Y}\ S_\alpha$,
3. $S_\alpha S_\beta\ \subseteq\ S_{\alpha\ \beta}$ for any $\alpha,\ \beta \in\ Y$,
4. $S_\beta \cap (S_\alpha] \neq \phi$ implies $\beta\ \preceq\ \alpha$, where $\preceq$ is the order of the semilattice Y defined by $\preceq := \{(\alpha,\ \beta)\ \mid\ \alpha = \alpha\ \beta\ (\beta\ \alpha)\}$ [11].

An element $a \in S$ is called ordered inverse of b if $a \leq aba$ and $b \leq bab$.

3 Scope of Work:

A regular semigroup(without order) is inverse if and only if every element of it has unique inverse. From this observation and from the definition of ordered inverse it is very natural to study inverse ordered semigroup. But the question arise how one can tackle uniqueness in ordered semigroup. Further there is a vast scope to extend the notion of right inverse in ordered semigroups.

4 Plan of Work:

I want to investigate the structure of inverse ordered semigroups as follows:

(1) Representing inverse ordered semigroups as an ordered semigroup where any two inverses of an element are $\mathcal{H}-$related.

(2) A completely regular ordered semigroup is Clifford (left) if and only if it is inverse (left).

(3) Characterizing nil extensions inverse and left inverse semigroups.

(4) Characterizing different radicals of Green's relations on inverse ordered semigroups.

(5) Characterizing LR-inverse ordered semigroups.

5 Work Done:

In paper (1) we have introduced the notions of inverse and left inverse ordered semigroups. Here we have shown that a completely regular ordered semigroup is Clifford (left) if and only if it is (left) inverse. Furthermore we observed that a left inverse ordered semigroup is complete semilattice of left group like ordered semigroups if and only if $\mathcal{L} = \mathcal{R}$.

6 Papers Communicated:

1) A. Jamadar and K. Hansda: On inverse and left inverse ordered semigroups.

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